

[2 pts] Question 1

Suppose that a jar contains 5 marbles, one each of the colors blue, green, purple, red, and yellow. Suppose now that marbles are drawn from the jar one at a time and immediately replaced. As marbles are drawn, their colors are tallied--we are just keeping count of how many times each color is drawn. The process is repeated 15 times, and the resulting counts are reported. How many different results are possible?

[3 pts] Question 2

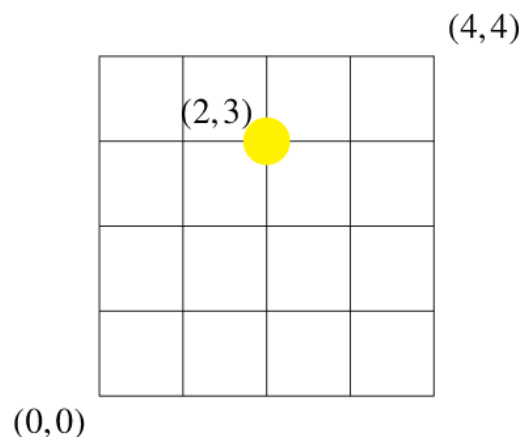
Suppose a computer program takes a sequence of letters without spaces (for example, a URL), and finds where the spaces go, by testing all possible ways of breaking up the sequence into segments and then checking the segments against a dictionary to determine whether each is a word. Given a sequence that is 20 characters long, how many ways are there to break the sequence into 3 nonempty segments?

[3 pts] Question 3

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 120$ if each x_i is an integer greater than or equal to 2?

[2 pts] Question 4

How many lattice paths are there from $(0, 0)$ to $(4, 4)$? How many of these paths pass through the point $(2, 3)$?

[2bnpts] Bonus

Give a combinatoric proof of Vandemonde's Identity.

$$\sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$$